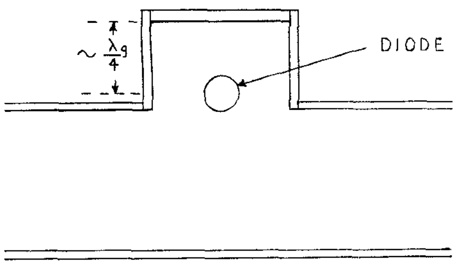

Fig. 1. Layout of H-plane switch.

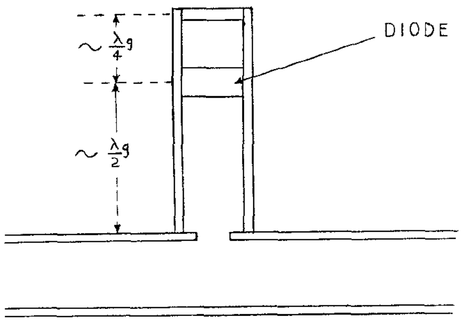

Fig. 2. Layout of recent switch.

A valid comparison also requires the use of similar diodes. The IN419 seems to be no longer available in the configuration used in 1961, but Philco switching diodes which are still available yielded similar results.

Because of its simplicity (Fig. 1) the earlier switch offers the advantages of 1) comparatively little measurement data or calculation required for its design; 2) good matching almost everywhere in the band for the "pass" condition, which comes about from the close placement of the diode to the junction (wider bandwidth of attenuation, as much as 2 per cent, can also be obtained from this feature); and 3) the ability to attenuate any frequency in the band, as determined solely by the position of the short. This switch is, in fact, most often used with a sliding short, so that the operating frequency can be changed at will. Diode placement favors the use of an H-plane tee for this switch, while the recent switch (Fig. 2) could supposedly be built equally well in the E or H plane.

An evident disadvantage is that the early switch must operate into a better match because of its close involvement in the junction. But because of these considerations, it is recommended that this switch be considered in applications using the recent design, especially in laboratory work, where flexibility is often desirable.

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Author's Comment[4]

Although our narrow-band waveguide switch[1] is of the same generic type (a band reject design in waveguide using diodes) as the one previously described, it is difficult to envisage any two switches in this category as different as these, both electrically and mechanically. The most striking difference in performance is evident from our Fig. 6, where it is shown that the isolation curve shifts by about 120 Mc/s for a change in diode state. According to Rebsch's correspondence of 1961,[2] the isolation of his switch is only present when the diode is forward biased. Perhaps Rebsch can be more specific about the electrical similarity he sees in these switches.

The central aim of our paper was to present a switch design which parallels the synthesis of passive band reject filters, and which therefore could be used in the design of switches for a wide variety of applications. By this procedure many of the difficult impedance matching problems associated with other types of switches are avoided. The degree to which this aim has been achieved is evident from our Figs. 6 to 8 which show a comparison of measured and computed responses as an example of the effectiveness of the synthesis procedure. It is significant that our design procedure is quite insensitive to the precise nature of the diode (and mount) impedance in either of the two states.

We agree that Rebsch's design presents matching problems, especially in the design of cascaded stages for the pass condition.

His other comments are rather general and therefore difficult to answer specifically, but we suggest that most engineers do not mind making calculations and measurements if the results are predictable. As a matter of fact, to be able to do so, is a rather delightful experience not encountered as often as one would like.

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[4] Manuscript received June 25, 1965.

Acceptable Mode Types for Inhomogeneous Media

This correspondence is prompted by Holmes's paper[1] in which he presented a study of the use of the WKB approximation for the solution of the wave equations in a rectangular waveguide inhomogeneously and continuously loaded across the broad dimension. I would like to point out the form of the acceptable mode types in such a waveguide.

Maxwell's equations are

$$\overline{E} = -j\omega\mu\overline{H} \quad (1)$$

$$\overline{H} = j\omega\epsilon\overline{E}. \quad (2)$$

Assume that μ and ϵ are functions of position. Taking the curl of (2) and substituting (1) we get

$$\nabla \times \nabla \times \overline{H} = \nabla \ln \epsilon \times (\nabla \times \overline{H}) + k^2\overline{H} \quad (3)$$

Manuscript received February 23, 1965.

[1] D. A. Holmes, "Propagation in rectangular waveguide containing inhomogeneous, anisotropic dielectric," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 152–5, March 1964.

where

$$\nabla \ln \epsilon = \frac{\nabla\epsilon}{\epsilon} \quad (4)$$

and

$$k^2 = \omega^2\mu\epsilon. \quad (5)$$

Since

$$\nabla\cdot\overline{B} = \nabla\mu\cdot\overline{H} + \mu\nabla\cdot\overline{H} = 0, \quad (6)$$

then

$$\nabla\cdot\overline{H} = -\overline{H}\cdot\nabla \ln \mu \quad (7)$$

with

$$\nabla \ln \mu = \frac{\nabla\mu}{\mu}. \quad (8)$$

As a result, the left-hand side of (3) may be written

$$\nabla \times \nabla \times \overline{H} = \nabla(\nabla\cdot\overline{H}) - \nabla^2\overline{H} = -\nabla(\overline{H}\cdot\nabla\ln\mu) - \nabla^2 H \quad (9)$$

and (3) becomes

$$(\nabla^2 + k^2)\overline{H} = (\nabla \times \overline{H}) \times \nabla \ln \epsilon - \nabla(\overline{H}\cdot\nabla \ln \mu). \quad (10)$$

Similarly,

$$(\nabla^2 + k^2)\overline{E} = (\nabla \times \overline{E}) \times \nabla \ln \mu - \nabla(\overline{E}\cdot\nabla \ln \epsilon). \quad (11)$$

Equations (10) and (11) are the wave equations for inhomogeneous media.

Consider now a rectangular waveguide in which μ and/or ϵ are functions of y only. As a result,

$$\nabla \ln \epsilon = \bar{u}_y \frac{\partial}{\partial y} \ln \epsilon \quad (12)$$

$$\nabla \ln \mu = \bar{u}_y \frac{\partial}{\partial y} \ln \mu. \quad (13)$$

Equations (14), (15), and (16) are the components of (10) when expanded in the rectangular coordinate system.

$$(\nabla^2 + k^2)H_x = \frac{\partial}{\partial y} H_x \frac{\partial}{\partial y} \ln \epsilon - \frac{\partial}{\partial x} H_y \frac{\partial}{\partial y} \ln(\mu\epsilon) \quad (14)$$

$$(\nabla^2 + k^2)H_y = -\frac{\partial}{\partial y} H_y \frac{\partial}{\partial y} \ln \mu - H_y \frac{\partial^2}{\partial y^2} \ln \mu \quad (15)$$

$$(\nabla^2 + k^2)H_z = \frac{\partial}{\partial y} H_z \frac{\partial}{\partial y} \ln \epsilon - \frac{\partial}{\partial z} H_y \frac{\partial}{\partial y} \ln(\mu\epsilon). \quad (16)$$

Expressions for the components of (11) are identical in form and, with the obvious substitutions, the following argument holds for electric fields as well.

Take x and y as the transverse coordinates and z as the direction of propagation. We now look for modes in which one of the components of magnetic field is nonexistent. The possibilities are as follows.

$H_z = 0$

From (16),

$$\frac{\partial}{\partial z} H_y \frac{\partial}{\partial y} \ln(\mu\epsilon) = 0. \quad (17)$$

This can occur when

1) $\mu\epsilon$ = constant, being a trivial case.
2) $H_y \neq f(z)$. Impossible for a wave

travelling in the z direction unless $H_y=0$ as well as H_z. This implies a TEM wave which cannot exist in an enclosed region. At any rate, consideration of the electric fields of a TEM wave in such an inhomogeneous medium will show that E_y must also disappear. Only E_x and H_x are then left and that is an impossible solution.

$H_z=0$

From (14),

$$\frac{\partial}{\partial x} y \frac{\partial}{\partial y} \ln (\mu\epsilon) = 0 \qquad (18)$$

which is satisfied when

1) $\mu\epsilon$=constant (as the preceding).
2) $H_y \neq f(x)$. This is a quite acceptable, although peculiar, case corresponding to solutions of the TE_{0n} type with electric field along x. $H_y=0$ also satisfies (18) but, as H_z is missing, this is an impossible solution of Maxwell's equations inside a waveguide. The analogous study of (11) leads to $E_y \neq f(x)$ which means that $E_y=0$. This indicates again that TE_{0n} modes are acceptable. (Notice that a TEM wave travelling in the y direction would be permissible on an open structure.)

$H_y=0$

Equation (15) is identically zero. The wave equations for the magnetic fields are now

$$(\nabla^2 + k^2)H_x = \frac{\partial}{\partial y} H_x \frac{\partial}{\partial y} \ln \epsilon \qquad (19)$$

$$(\nabla^2 + k^2)H_z = \frac{\partial}{\partial y} H_z \frac{\partial}{\partial y} \ln \epsilon \qquad (20)$$

which can have no inconsistencies between them. If the solution for $\overline{H}$ can be found, then the electric field is given by (2).

The result of the preceding analysis is that longitudinal-section (LS) modes[2] ($E_y=0$ or $H_y=0$) are shown to be the normal ones for such a waveguide as they are the only ones not requiring the stipulation of impossible or restricting conditions. Unfortunately, Holmes has developed his theory in terms of TM or TE modes (i.e., $H_z=0$ and $E_z=0$, respectively). There is no apparent reason, however, that the theory cannot be easily altered. That part of the paper dealing with TE_{0n} modes (which are a particular type of LS mode with $E_y=0$) is apparently correct.

By an argument almost identical to that which has gone before, it follows that LS modes are inadmissible solutions when $\mu\epsilon=f(z)$. In this case, TM and TE modes are required. Therefore, TE_{0n} modes, which are of both the TE and LS types, are valid for waveguides in which $\mu\epsilon$ is a function of the direction of propagation and of one transverse coordinate.

A slab-loaded waveguide represents the limiting case of a rapid but finite change of, say, dielectric constant. It has long been known[3] that one cannot choose just any mode type and attempt to match transverse components at the boundary without the danger of inconsistencies in deriving the eigenvalue equation. Matching only one electric and one magnetic component across the interface does not ensure that the other transverse components are likewise matched. The reason for such inconsistencies in boundary matching is evident. The regions being considered are not simply the dielectric and air ones. There is another—the transition region—and modes must he chosen that are appropriate to it as well. When such a transition region is of vanishingly small extent, the retardation across it is negligible and the tangential fields (or normal fluxes) may be equated through it. This approach obviates the necessity of solving the inhomogeneous wave equation explicitly. In spite of this simplification, the normal modes must still be of the type, at least across the inhomogeneous region.

It was rather optimistically thought that, as a direct extension of the rectangular waveguide results, modes of the form $E_r=0$ and $H_r=0$ would prove satisfactory for a radially inhomogeneous circular guide. Although it is impossible to find modes with a missing radial component (except for the TE_{0n} and TM_{0n}), a linear superposition of degenerate TE and TM waves can be formed in such proportions that either of the radial components can be made to vanish at any chosen radius r_0. Then, it was hoped that these new modes would be solutions for a circular guide loaded with a rod of that radius.

For a radially inhomogeneous guide,

$$\nabla \ln \epsilon = \bar{u}_r \frac{\partial}{\partial r} \ln \epsilon \qquad (21)$$

and

$$\nabla \ln \mu = \bar{u}_r \frac{\partial}{\partial r} \ln \mu. \qquad (22)$$

Expanding (10) in cylindrical coordinates and separating components, several possibilities can be studied.

$H_r=0$

We find, from the expression for the r component, that

$$\frac{\partial}{\partial \phi} H_\phi = 0. \qquad (23)$$

Therefore

$$H_\phi \neq f(\phi). \qquad (24)$$

Only the TM_{0n} mode satisfies both (24) and $H_r=0$. This shows immediately that a complete set cannot be formed in this way. The other cases follow.

$H_\phi=0$

From the ϕ-component expression,

$$\frac{2}{r} \frac{\partial}{\partial \phi} H_r = - \frac{\partial}{\partial \phi} H_r \frac{\partial}{\partial r} \ln (\mu\epsilon). \qquad (25)$$

This holds for

1) TE_{0n} modes and when
2) $\mu\epsilon r^2$ (or kr) is a constant. This places a restriction on the loading.

$H_z=0$

The z component gives

$$\frac{\partial}{\partial z} H_r \frac{\partial}{\partial r} \ln (\mu\epsilon) = 0 \qquad (26)$$

which is true for

1) TM_{0n} modes.
2) $\mu\epsilon$-constant. This is the trivial case of uniform loading.

A similar analysis of (11) will give identical results if, in the preceding TM_{0n} and TE_{0n} are interchanged and the components of $\overline{E}$ are substituted for those of $\overline{H}$. Then, since five-component modes are in general unsatisfactory, it is clear that six-component hybrid modes must be used. Hybrid modes are also found to be required for circumferential inhomogeneities, again with the exception of the TM_{0n} and TE_{0n} wave types. For z inhomogeneities, TM and TE modes are acceptable.

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D. A. Holmes[4]

I particularly appreciate and welcome Wexler's articulate and rather adroit comments because they permit me to compensate for the brevity from which my paper[1] suffers. In the first place, I wish to point out that the results presented in that paper were derived for a nearly uniaxial material, i.e., $K_x(x) \cong K_y(x)$. Upon re-examination, I find that the distinction between $K_x(x)$ and $K_y(x)$ is not worth retaining. I suggest that the previously derived solutions[1] be applied only to a strictly uniaxial material oriented such that the optic axis is parallel to the direction of propagation. This can be accomplished by using the definitions[5]

$$K_x(x) = K_y(x) \equiv K_o(x), \quad K_z(x) \equiv K_e(x), \quad (27)$$

where $K_o(x)$ and $K_e(x)$ can be called the ordinary and extraordinary dielectric constants, respectively. In keeping with (27), I shall consider a uniaxial material in the work to follow.

As a prelude to further discussion, consider the following mathematical development. By assuming $\exp(j\omega t - \Gamma z)$ variations in all electric and magnetic field components, the Maxwell curl equations can be written in the form

$$gE_y = -\Gamma(\partial E_z/\partial y) + j\omega\mu_0(\partial H_z/\partial x), \qquad (28)$$

$$gE_x = -\Gamma(\partial E_z/\partial x) - j\omega\mu_0(\partial H_z/\partial y), \qquad (29)$$

$$gH_y = -\Gamma(\partial H_z/\partial y) - j\omega\epsilon_0 K_o(x)(\partial E_z/\partial x), \qquad (30)$$

$$gH_x = -\Gamma(\partial H_z/\partial x) + j\omega\epsilon_0 K_o(x)(\partial E_z/\partial y), \qquad (31)$$

$$\partial E_y/\partial x - \partial E_x/\partial y = -j\omega\mu_0 H_z, \qquad (32)$$

$$\partial H_y/\partial x - \partial H_x/\partial y = j\omega\epsilon_0 K_e(x) E_z, \qquad (33)$$

where

$$g = \Gamma^2 + k_0^2 K_0(x).$$

[2] R. F. Harrington, *Time-Harmonic Electromagnetic Field*. New York: McGraw-Hill, 1961, pp. 152–155.

[3] L. Pincherle, "Electromagnetic waves in metal tubes filled longitudinally with two dielectrics," *Phys. Rev.*, vol. 66, pp. 118–30, September 1 and 15, 1944.

[4] Manuscript received April 12, 1965.

[5] Except when specified otherwise, my notation here is the same as that used in my earlier work.[1]

By *assuming* that

$$H_z = 0, \quad E_z = E\psi_{TM}(x)\cdot \sin(m\pi y/b), \tag{34}$$

and substituting (34), (30), and (31) into (33) we obtain

$$\frac{1}{\alpha_{TM}}\cdot\frac{\partial}{\partial x}\left\{\alpha_{TM}\frac{\partial \psi_{TM}}{\partial x}\right\} + \beta_{TM}^2\psi_{TM} = 0, \tag{35a}$$

where

$$\alpha_{TM} = \frac{K_o(x)}{\Gamma_{TM}^2 + k_0^2 K_o(x)}, \tag{35b}$$

$$\beta_{TM}^2 = \frac{K_e(x)}{\alpha_{TM}} - \left(\frac{m\pi}{b}\right)^2. \tag{35c}$$

By *assuming* that

$$E_z = 0, \quad H_z = H\psi_{TE}(x)\cdot \cos(m\pi y/b), \tag{36}$$

and substituting (36), (28), and (29) into (32) we obtain

$$\frac{1}{\alpha_{TE}}\cdot\frac{\partial}{\partial x}\left\{\alpha_{TE}\frac{\partial \psi_{TE}}{\partial x}\right\} + \beta_{TE}^2\psi_{TE} = 0, \tag{37a}$$

where

$$\alpha_{TE} = [\Gamma_{TE}^2 + k_0^2 K_o(x)]^{-1}, \tag{37b}$$

$$\beta_{TE}^2 = \frac{1}{\alpha_{TE}} - \left(\frac{m\pi}{b}\right)^2. \tag{37c}$$

Although I have attached TE and TM subscripts to various quantities in (34)–(37), the preceding development clearly does not, in itself, justify a decomposition into TE and TM modes. When (34)–(37) are used to obtain the solutions listed in Sections II and III of my previous paper,[1] using the definitions (27), it is found that the listed solutions do not rigorously satisfy the Maxwell curl and divergence equations. Imbedded in the WKB approximation, however, is the restriction that $K_o(x)$ and $K_e(x)$ are *slowly varying* functions of x. It is of interest to discuss the logical limit of the slow variation case, namely, that of a homogeneous medium for which $K_o(x) = K_o$, $K_e(x) = K_e$, where K_o and K_e are constants. For a homogeneous uniaxial medium wholly filling the rectangular waveguide, a decomposition into TE and TM modes is valid; for this case we find

$$\psi_{TE} = \cos(n\pi x/a), \tag{38a}$$

$$\psi_{TM} = \sin(n\pi x/a), \tag{38b}$$

$$\Gamma_{TE}^2 = (m\pi/b)^2 + (n\pi/a)^2 - k_0^2 K_o, \tag{38c}$$

$$\Gamma_{TM}^2 = [(m\pi/b)^2 + (n\pi/a)^2]\cdot(K_o/K_e) - k_0^2 K_o. \tag{38d}$$

The electric and magnetic fields found from (38) satisfy the boundary conditions for perfectly conducting walls and satisfy all of the Maxwell equations.

In approaching the slow variation case, my reasoning was more physical than mathematical. In light of the preceding discussion of the homogeneous case, it seems entirely reasonable that propagation in a slightly inhomogeneous medium can be classified into a mode structure which is nearly TE or TM. Based on this physical reasoning, my procedure was to *search* for TE and TM modes with the results (34)–(37). WKB solutions never rigorously satisfy the differential equations from whence they came, however, the particular solutions given by me[1] do satisfy the boundary conditions at the waveguide walls and represent approximations which become exact in the homogeneous limit.

I feel that the general area of slight inhomogeneities or slow variations is a "no man's land" which, undoubtedly, may be approached on many trajectories. For the specific problem which I have considered, I cannot claim, at this time, that my treatment is more general than, or preferable to, an alternative approach.[6] My contribution is that the TE and TM mode WKB approach provides *predictions* which the experimentalist can test in the laboratory.

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[6] Observe that Wexler's discussion of longitudinal-section electric (LSE) and longitudinal-section magnetic (LSM) mode propagation in an isotropic medium would require modification in order to allow inclusion of a uniaxially anisotropic medium.

A. Wexler[7]

Holmes is quite correct in noting that my comments apply to isotropic media and require modification if an anisotopic medium is considered. To follow his suggestion, one interesting case will now be investigated. The components of the tensor permittivity are taken as defined in (27). The permeability is taken to be a constant scalar at the free-space value μ_0. As I previously assumed the medium to be a function of y, this correspondence will continue in that way. Holmes takes it to vary along x but this should cause no confusion.

Taking the curl of (1), substituting (2), and rewriting the left-hand side of the wave equation, we obtain

$$\nabla(\nabla\cdot\overline{E}) - \nabla^2\overline{E} = k_0^2(\bar{u}_x K_o(y)E_x + \bar{u}_y K_o(y)E_y + \bar{u}_z K_e(y)E_z). \tag{39}$$

Since $\nabla\cdot\overline{D} = 0$, we find that

$$K_o(y)\frac{\partial E_x}{\partial x} + K_o(y)\frac{\partial E_y}{\partial y} + E_y\frac{\partial K_o(y)}{\partial y} + K_e(y)\frac{\partial E_z}{\partial z} = 0. \tag{40}$$

The first two terms of (40) can be substituted into the expansion of $\nabla\cdot\overline{E}$, giving

$$\nabla\cdot\overline{E} = (1 - K_e(y)/K_o(y))\frac{\partial E_z}{\partial z} - (E_y/K_o(y))\frac{\partial K_o(y)}{\partial y}. \tag{41}$$

By substituting (41), the y component of (39) is found in a convenient form. When this is done we may put $E_y = 0$, producing the following condition for the existence of longitudinal-section electric (LSE) modes:

$$\frac{\partial}{\partial y}\left[(1 - K_e(y)/K_o(y))\frac{\partial E_z}{\partial z}\right] = 0. \tag{42}$$

Equation (42) holds when $E_z = 0$. And so, TE_{0n} modes are of a legitimate type in this uniaxially anisotropic medium. The other possibility, when the ratio $K_e(y)/K_o(y)$ is constant over the cross section, allows for the existence of all LSE modes. Otherwise it seems that six-component hybrid modes are generally required.

[7] Manuscript received June 25, 1965.

Holmes's theory, whose importance lies in its simplicity and usefulness, would become significantly less convenient if hybrid modes were employed. As he has already shown, transverse-electric and magnetic modes can exist in a homogeneous, uniaxial medium and—since the WKB solution is a slow-variation approximation to begin with —it is entirely reasonable to use them to study slight inhomogeneities. However, it is not difficult to see that LS modes can also exist under these conditions.

Assume the z component of electric field to be

$$E_z = A\sin k_x x\cdot\sin k_y y \tag{43}$$

where

$$k_x = n\pi/a \tag{44}$$

and

$$k_y = m\pi/b. \tag{45}$$

Taking $H_y = 0$, we find from (30) that

$$\frac{\partial H_z}{\partial y} = \frac{-j\omega\epsilon_0 K_o}{\Gamma}\cdot\frac{\partial E_z}{\partial x} = \frac{-j\omega\epsilon_0 k_x K_o}{\Gamma}A\cos k_x x\cdot\sin k_y y. \tag{46}$$

Integrating,

$$H_z = \frac{j\omega\epsilon_0 k_x K_o}{\Gamma k_y}A\cos k_x x\cdot\cos k_y y. \tag{47}$$

The other term which results from the integration of (46) is not a function of y and can be discounted on physical grounds. Since E_z and H_z are now both known, the rest of the field components may be derived from (28) to (31). Substituting the appropriate components into (32), (38c) results and completes the description of the longitudinal-section magnetic (LSM) modes.

If now we take $E_y = 0$, we find the following from (28) and (43):

$$H_z = \frac{j\Gamma k_y}{\omega\mu_0 k_x}A\cos k_x x\cdot\cos k_y y. \tag{48}$$

The remaining field components of the LSE set may be derived easily. Substituting into (33), (38d) is found.

As a result of the preceding discussion, it is evident that LS modes are allowable solutions in a rectangular waveguide completely filled with a homogeneous, uniaxial medium. By the same physical reasoning that Holmes used, solutions in a slightly inhomogeneous medium could equally have been approximated by them. As these modes are the correct ones for an inhomogeneous, isotropic guide, their use instead of a TE or TM set should be at least marginally beneficial in this case. It seems reasonable to expect that in this way Holmes's theory should give better answers, as the inhomogeneity increases, than might otherwise be obtained. This presumed advantage may turn out to be of negligible practical importance but it is worth bearing in mind.

As we are bordering on the subject, I would like to carry this correspondence just a little further in order to point out an apparent anomaly in the use of hybrid modes.

In addition to the problem just discussed, six-component hybrid modes have been found to be necessary for general solutions in ferrite-loaded rectangular waveguides[8,9] and in rod-loaded, dielectric or ferrite, circular guides.[3,10] Hybrid mode solutions have been found by taking linear combinations of TE and TM modes in each region and matching all fields parallel and fluxes normal to the discontinuity; in practice, it is necessary to match only four components to achieve this. To check this method, we will compare the results obtained in a particular problem which is solved by both hybrid and LS modes.

Consider an isotropic, rectangular waveguide divided into two transverse regions having different dielectric and magnetic properties. The interface, in the x-z plane, occurs at $y=c$ and the guide sidewalls are at $y=0$ and $y=b$. Propagation is in the z direction. The TE and TM solutions are known in each region and are to be combined in proportions to be determined by matching considerations. There are four unknown amplitude constants. By matching four field components across the boundary, the unknown constants may be eliminated, resulting in an eigenvalue equation.

Matching E_z, H_z, E_x, and H_x at $y=c$ we find after some algebraic manipulation, that

$$\epsilon_1 k_{y1}(k_x^2+k_{y2}^2)\cdot\cot k_{y1}c+\epsilon_2 k_{y2}(k_x^2+k_{y1}^2)\cdot\cot k_{y2}(b-c) = \frac{k_x^2 k_z^2(k_2^2-k_1^2)(\mu_1\epsilon_1-\mu_2\epsilon_2)}{\mu_1 k_{y1}(k_x^2+k_{y2}^2)\cdot\tan k_{y1}c+\mu_2 k_{y2}(k_x^2+k_{y1}^2)\cdot\tan k_{y2}(b-c)} \quad (49)$$

where the cutoff relation is

$$k_x^2+k_y^2+k_z^2=k^2 \quad (50)$$

and

$$k_z=-j\Gamma. \quad (51)$$

The subscripts 1 and 2, on k_y and k, refer to the regions in which $0<y<c$ and $c<y<b$, respectively. k_x and k_z must be the same in both regions so that the fields will be matched for all x and z. k_x is given by (44).

If now, E_z, H_z, E_x, and D_y are matched, (52) results.

$$\mu_1 k_{y1}(k_x^2+k_{y2}^2)\cdot\cot k_{y2}(b-c)+\mu_2 k_{y2}(k_x^2+k_{y1}^2)\cdot\cot k_{y1}c = \frac{k_x^2(k_2^2-k_1^2)\left[\mu_2\epsilon_2(k_x^2+k_{y1}^2)-\mu_1\epsilon_1(k_x^2+k_{y2}^2)\right]}{\epsilon_1 k_{y1}(k_x^2+k_{y2}^2)\cdot\tan k_{y2}(b-c)+\epsilon_2 k_{y2}(k_x^2+k_{y1}^2)\cdot\tan k_{y1}c}. \quad (52)$$

The eigenvalue equations (49) and (52) are both rather involved. We know, on firm theoretical grounds, that LS modes should give correct results. Therefore, using LSM and LSE modes respectively, the following are found:

$$\frac{\epsilon_2}{\epsilon_1}=-\frac{k_{y2}\cot k_{y1}c}{k_{y1}\cot k_{y2}(b-c)} \quad (53)$$

$$\frac{\mu_2}{\mu_1}=-\frac{k_{y2}\cot k_{y2}(b-c)}{k_{y1}\cot k_{y1}c}. \quad (54)$$

These hold for all k_x. To be sure that (52) is not the same as (54), a numerical solution of the latter was substituted into the former and found not to be satisfactory. This must almost certainly be true of (49) and (53) as well. Even though the fields are completely matched in all cases, the eigenvalue equations obtained are inconsistent!

As a variant of the previous hybrid approach, six-component hybrid modes were formed by taking combinations of LSE and LSM modes (where before, TE and TM pairs were used). First of all, D_y and B_y were matched. Then, when E_x and E_z were matched, (53) resulted; when H_x and H_z were matched, (54) was obtained. No disagreement appears here as the eigenvalue equations turn out correctly.

It is difficult to believe that (49) and (52) are completely wrong. After all, they were derived from modes that individually satisfied Maxwell's equations in the homogeneous regions and, taken in pairs, the internal boundary as well. Note that (53) and (54) result from (49) and (52) when $k_x=0$ is substituted. Although this condition is not physically realizable for LSM modes and is unduly restricting for LSE modes, it suggests that there are particular values of k_x (not necessarily zero) for which correct solutions of (49) and (52) may be found. It is likely that these roots correspond to a restricted range of solutions and therefore furnish incomplete sets. Examination may reveal that all the roots of (49) satisfy (53) although, as was evident from the numerical example, the converse is not always true.

Because of the questionable result obtained, there is a good case for a critical reexamination of the derivation and use of hybrid modes. There is something unsatisfactory about the present method and, unless the example given is fallacious, a number of similar studies will have to be reconsidered.

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Measurement of Cutoff Frequencies

Measurements of guide wavelengths and cutoff frequencies are often of interest in experimental investigations of waveguides with general cross sections. When necessary measuring equipment is not available, the phase constant measurement described by Altschuler[1] provides a simple method of determining the cutoff frequency, which is readily obtainable from the phase constant of the waveguide. However, if a more accurate determination is required, the preceding method may not be adequate. A more accurate and convenient method for determining cutoff frequencies is presented here.

It is assumed that the waveguide under consideration is uniform, cylindrical, and with an arbitrary cross section. It is possible to form a resonant cavity by shorting both ends of a guide. Through an iris in one of the shorts, energy is fed into the cavity, and resonance is observed using a directional coupler and a detector setup as shown in Fig. 1. Let f_0 and f_q be two resonant frequencies of the cavity with f_q larger than f_0. The two resonant modes must be in the same transverse variation with only the number of longitudinal variations differing by an integer q. In other words, if m and n represent the transverse variational numbers, and p represents that of the longitudinal, then f_0 is in the (m, n, p) mode, and f_q is in the $(m, n, p+q)$ mode.

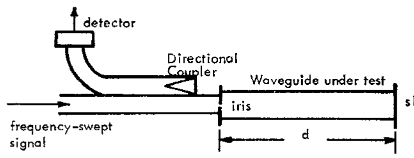

Fig. 1. Simplified diagram for observing cavity resonance.

The electrical length of a waveguide with two ideal shorts on both ends is a multiple of π radians. However, due to the coupling iris in one of the shorts, the length differs from the ideal case by a fraction δ.[2] At the resonant frequency f_0, the electrical length is expressed as follows:

$$\beta_0 d=(N-\delta_0)\pi \quad (1)$$

where β_0 is the phase constant, d is the physical length of the line, and N is an integer.

The physical length of the cavity at resonance is a multiple of half guide wavelengths. In a similar argument[2] the length is

$$d=(N-\delta_0)\lambda_0/2 \qquad \text{at } f_0 \quad (2)$$

and

$$d=(N-\delta_q+q)\lambda_q/2 \qquad \text{at } f_q \quad (3)$$

where λ_0 and λ_q are the guide wavelengths at f_0 and f_q, respectively. Using (1)–(3) and eliminating N results into

$$2d=(q+\delta_0-\delta_q)\lambda_q\lambda_0/(\lambda_0-\lambda_q). \quad (4)$$

If the frequency range of operation is not too wide, the fractions δ_0 and δ_q are almost equal, as will be demonstrated later. Equation (4), therefore, reduces to

$$q/2d=1/\lambda_q-1/\lambda_0 \quad (5)$$

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[2] H. M. Barlow and A. L. Cullen, *Microwave Measurements*. London, England: Constable and Co. Ltd., 1950, ch. 3.